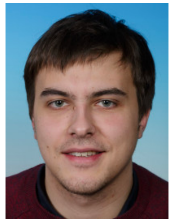
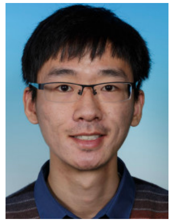
Quantum Non-Demolition measurement of a many-body Hamiltonian

Denis Vasilyev





A Grankin L Sieberer D Yang M Baranov P Zoller

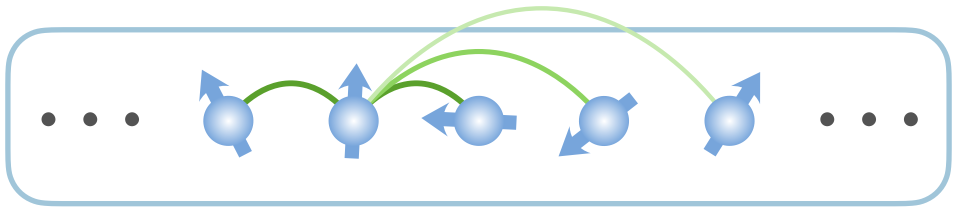
Quantum Non-Demolition measurement of a many-body Hamiltonian

Denis Vasilyev

- `*Single shot*' measurement of `*the energy*' of a quantum many-body system

Introduction

Analog quantum simulator



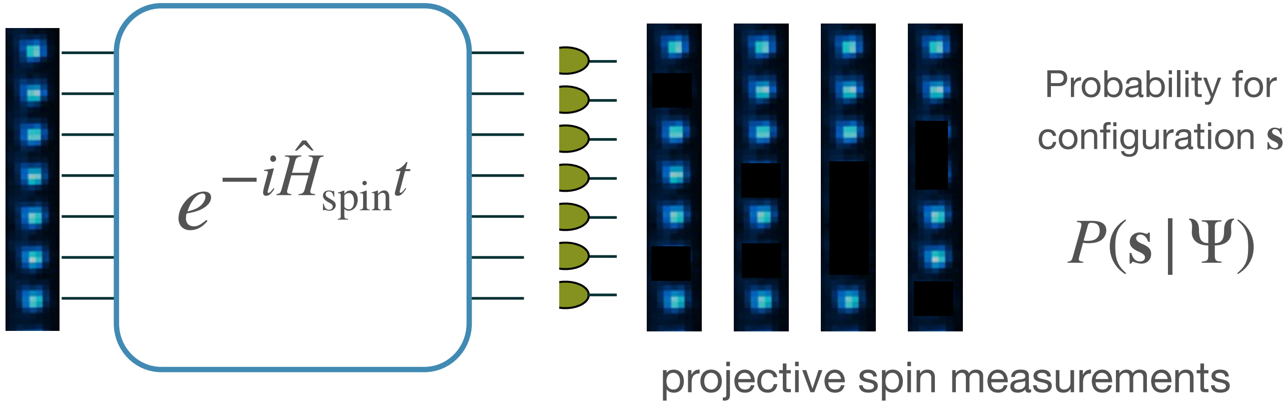
Trapped Ions
Rydberg Tweezer Arrays

Nat. Phys. 16, 132 (2020)

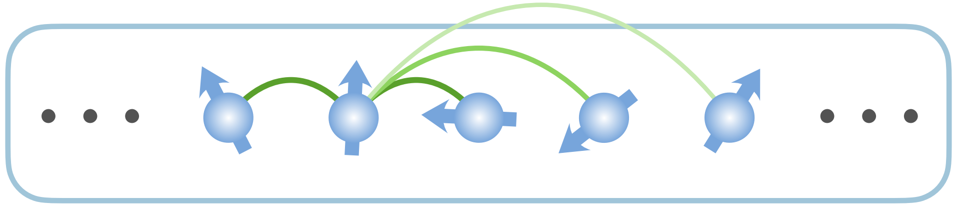
Nature 567, 61–65 (2019)

The standard scenario of analog quantum simulator

- A broad class of many-body Hamiltonians H_{spin} can be designed
- Correlation functions are inferred from (destructive) site-resolved readout of spins



Analog quantum simulator



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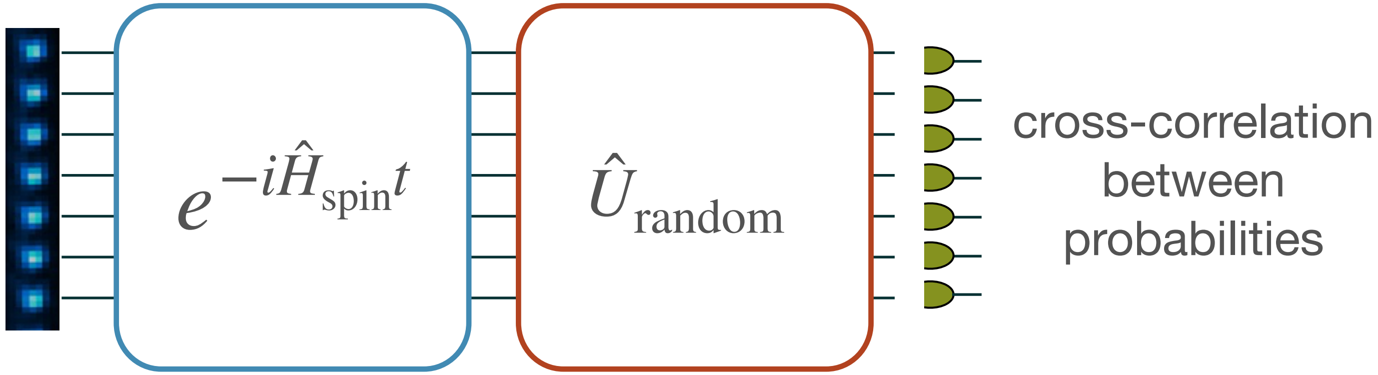


randomized measurement toolbox

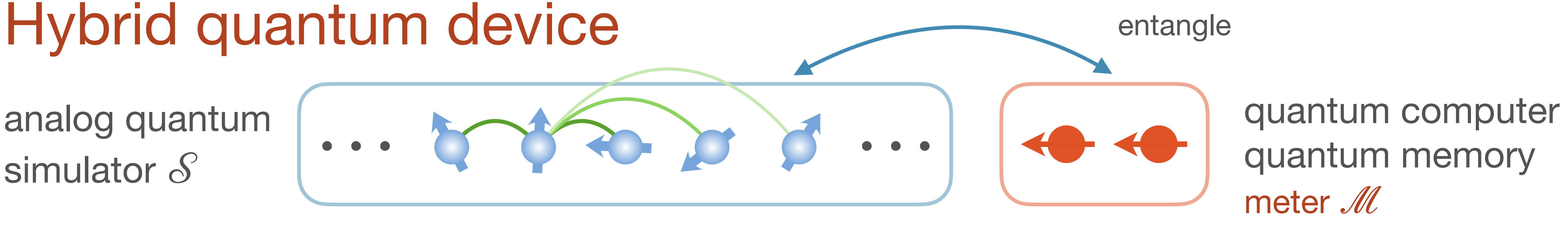
No build-in ‘energy’ meter

A. Elben, et. al., Sci. Adv. 6, (2020)

Why QND energy measurement?

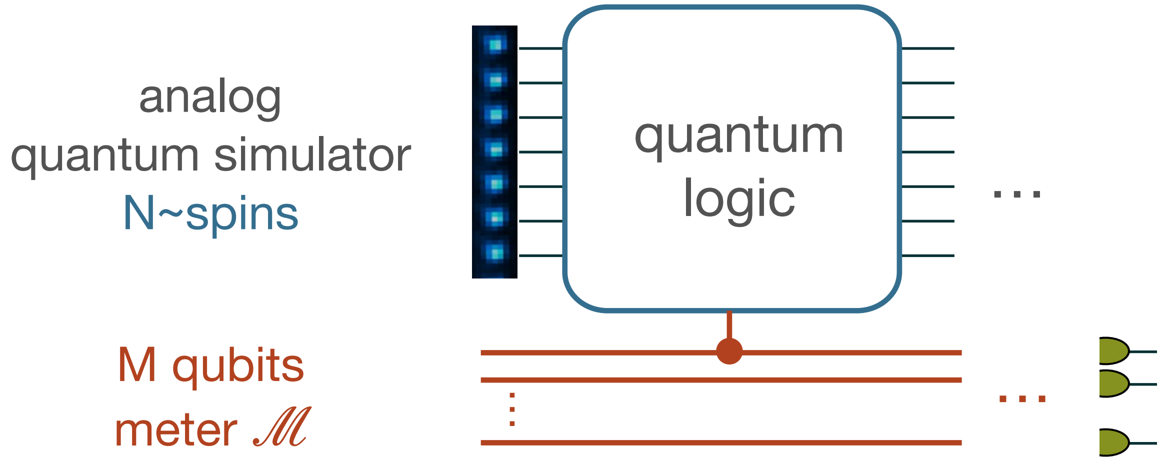
- In general, a many-body system has only one QND observable —
  the system Hamiltonian itself
- Energy is a fundamental physical quantity
- Quantum thermodynamics assumes the projective energy measurement for quantum fluctuations relations (Jarzynsky equality)
- Eigenstate thermalisation hypothesis
- Study of quantum chaos to MBL transition
- …

Hybrid quantum device

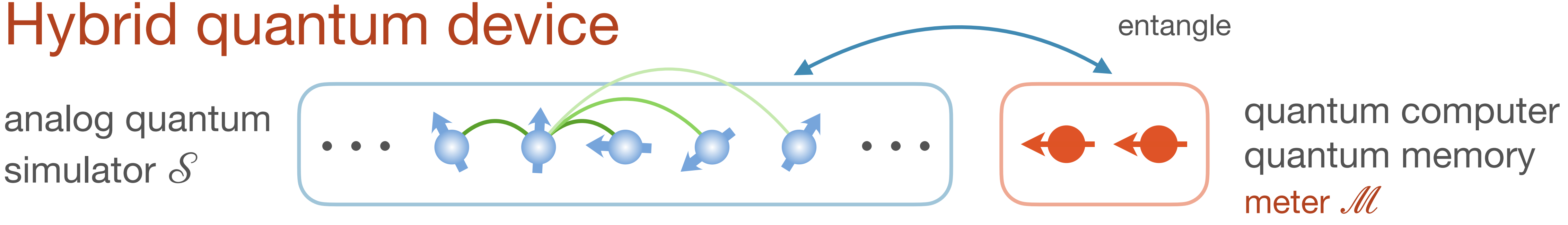


We are interested in a hybrid device: Quantum Computer (meter) + Quantum Simulator

- The many-body system is entangled with an auxiliary quantum system
- The auxiliary system (meter) is measured

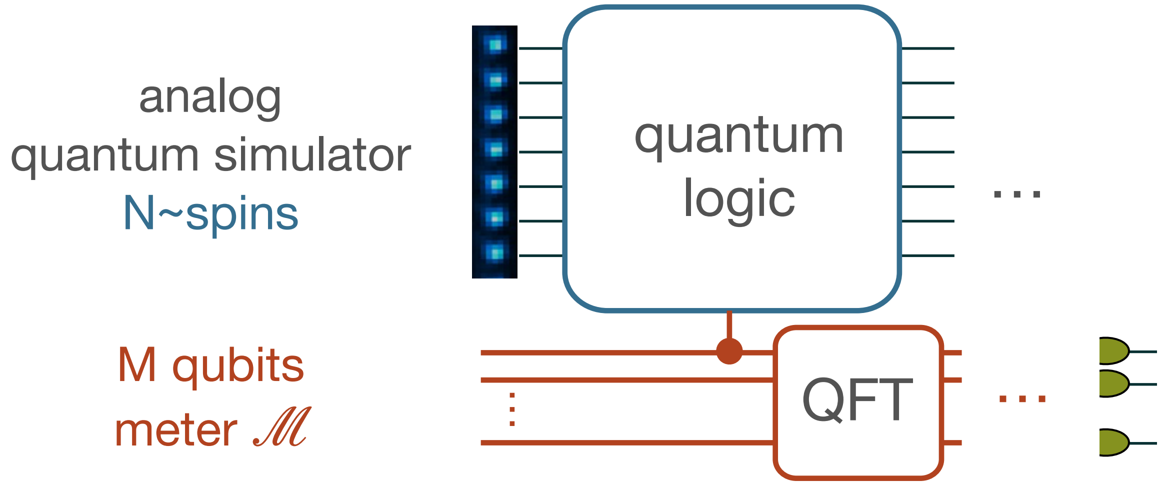


Hybrid quantum device



We are interested in a hybrid device: Quantum Computer (meter) + Quantum Simulator

- The many-body system is entangled with an auxiliary quantum system
- More generally the auxiliary system can be a small scale quantum computer (e.g. QFT)



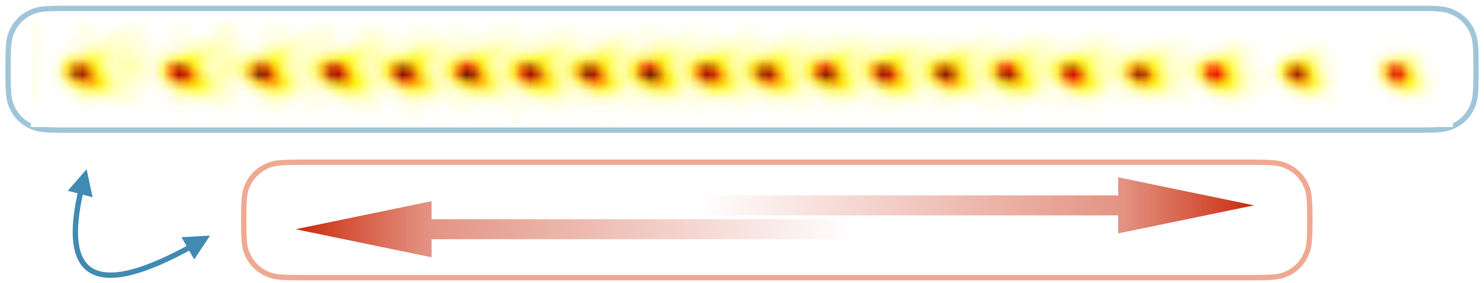
Prepare, manipulate, observe a quantum simulator with (small scale) quantum memory / computer possibly running simple quantum algorithms

Rydberg Tweezer arrays

DV et.al., PRX Quantum **1**, 020302 (2020)

Hybrid quantum device: Simulator + Continuous Variable Mode

analog quantum simulator $\mathscr{S}$



ions ~ spins

phonons ~ meter $\mathscr{M}$

entangle

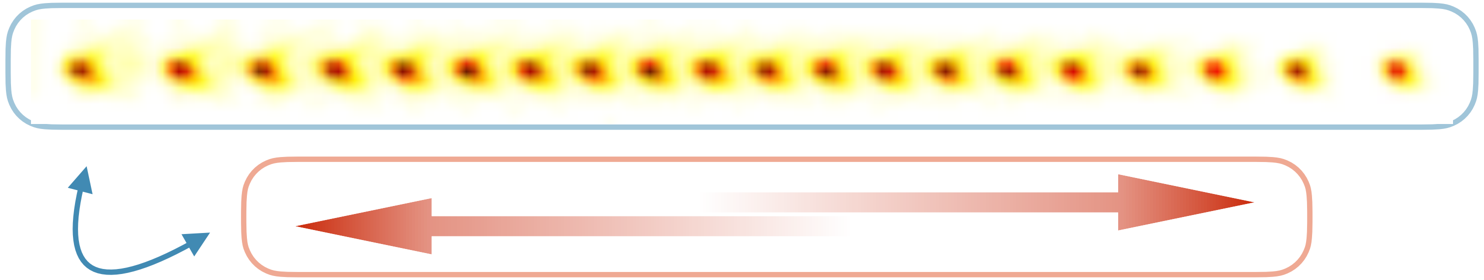
COM mode as continuous variable

This talk: Trapped Ions

Yang et.al., Nat Commun **11**, 775 (2020)

Hybrid quantum device: Simulator + Continuous Variable Mode

analog quantum simulator $\mathscr{S}$



ions ~ spins

phonons ~ meter $\mathscr{M}$

entangle

COM mode as continuous variable

$$[\hat{X}_M, \hat{P}_M] = i$$

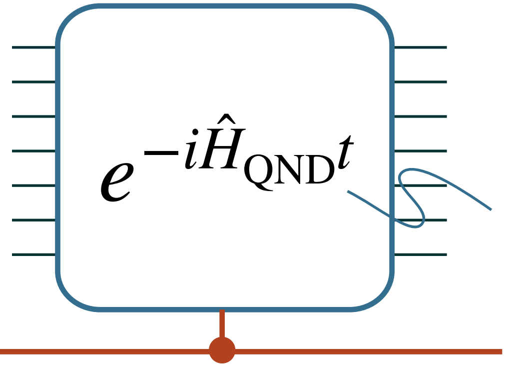
meter variable ~quadratures

Many-Body System coupled to a Continuous Meter

Building block: QND-quantum gate

generated by QND-Hamiltonian

Challenge is to implement the three body interaction



analog QND-gate

meter $\mathscr{M}$

$$\hat{H}_{\mathrm{QND}} = \vartheta\, \hat{H}_{\mathrm{spin}} \otimes \hat{P}_M$$

engineered spin-Hamiltonian $\mathscr{S}$

meter $\mathscr{M}$

This talk: Trapped Ions

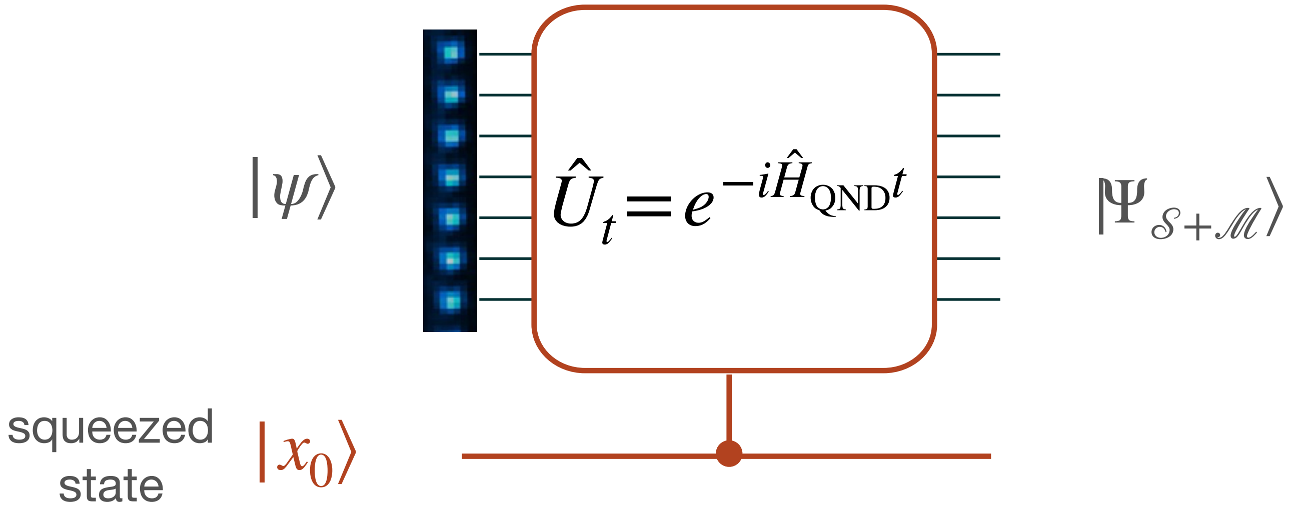
Yang et.al., Nat Commun **11**, 775 (2020)

QND-Measurement of $\hat{H}_{\text{spin}}$

QND-Measurement of $\hat{H}_{\text{spin}}$ — Step 1: Entangle System & Meter

Many-Body System coupled to a Meter

$$\hat{H}_{\text{QND}} = \hat{H}_{\text{spin}} \otimes \hat{P}_z$$



$$|\psi\rangle \otimes |x_0\rangle \xrightarrow{\text{evolve}} |\Psi_{\mathcal{S}+\mathcal{M}}\rangle = \sum_\ell c_\ell\, |\ell\rangle e^{-iE_\ell t} \otimes |x_0 + \vartheta E_\ell t\rangle$$

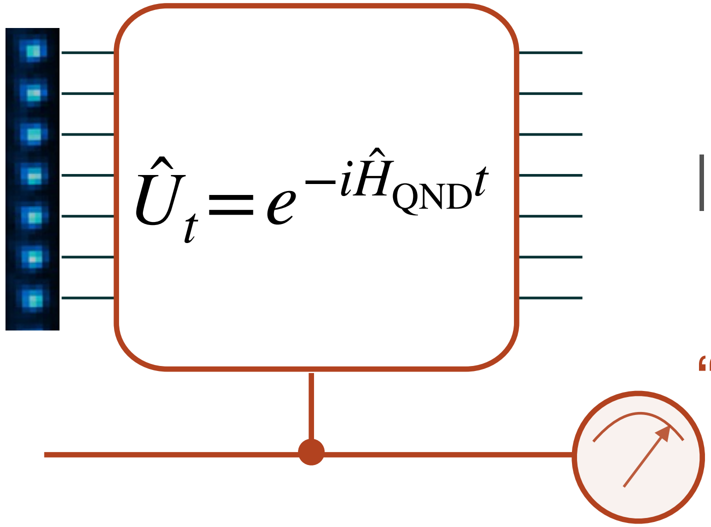
$$\equiv \sum_\ell c_\ell\, |\ell\rangle \otimes |x_0\rangle \quad \text{with} \quad \hat{H}_{\text{spin}}\, |\ell\rangle = E_\ell\, |\ell\rangle \quad \text{(energy eigenbasis)}$$

QND-Measurement of $\hat{H}_{\text{spin}}$ — Step 2: QND-Measurement

$$\hat{H}_{\text{QND}} = \hat{H}_{\text{spin}} \otimes \hat{P}_z$$

Many-Body System coupled to a Meter

$|\psi\rangle$



$|\ell\rangle$

squeezed state $|x_0\rangle$

"$x_\ell \sim E_\ell$"

QND-Measurement of $\hat{H}_{\text{spin}}$

- measuring the meter $x_\ell \sim \vartheta E_\ell$ reveals energy eigenvalue E_ℓ of $\hat{H}_{\text{spin}}$,
- prepares system in energy eigenstate $|\ell\rangle$
- which remains unchanged under repeated QND-measurements

$$|\psi\rangle \otimes |x_0\rangle \xrightarrow{\text{evolve}} |\Psi_{\mathcal{S}+\mathcal{M}}\rangle = \sum_\ell c_\ell |\ell\rangle e^{-iE_\ell t} \otimes |x_0 + \vartheta E_\ell t\rangle$$

$$\xrightarrow{\text{measure } \hat{X}_M\text{: “}x_\ell \sim E_\ell\text{”}} |\ell\rangle \otimes |x_0 + \vartheta E_\ell t\rangle$$

with probability:

$$\mathcal{P}_\ell = |c_\ell|^2$$

QND-Measurement of $\hat{H}_{\text{spin}}$ — Step 2: QND-Measurement

This is NOT measurement of the expectation value $\langle \hat{H}_{\text{spin}} \rangle$

QND-Measurement of $\hat{H}_{\text{spin}}$

- measuring the meter $x_\ell \sim \vartheta E_\ell$ reveals energy eigenvalue E_ℓ of $\hat{H}_{\text{spin}}$,
- prepares system in energy eigenstate $|\ell\rangle$
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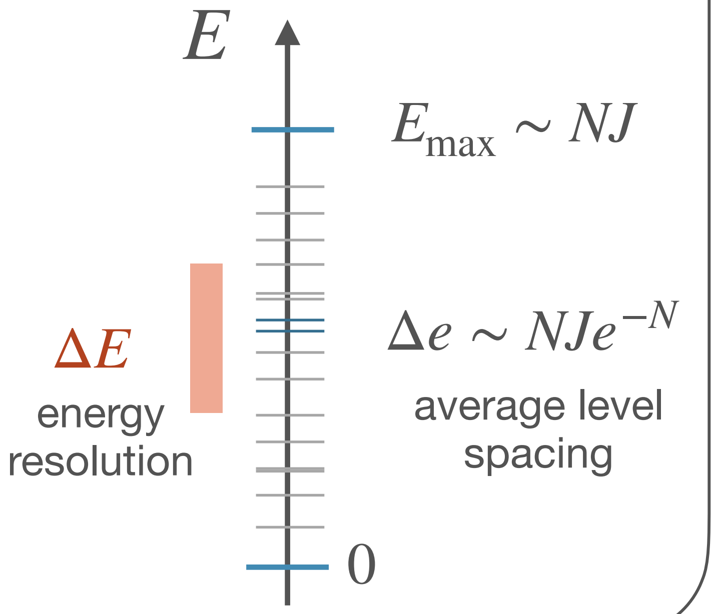
QND-Measurement of $\hat{H}_{\text{spin}}$ — Step 2: QND-Measurement

- **quantum N-body system dim $\sim 2^N$**

  resolve single vs. band of eigenstates

$$\hat{H}_{\text{spin}} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$



SQL $\Delta E \sim \dfrac{1}{\sqrt{T_{\text{mes}}}} \leftrightarrow \tau_{\text{decoherence}}$

Challenge

QND-Measurement of $\hat{H}_{\text{spin}}$

- measuring the meter $x_\ell \sim \vartheta E_\ell$ reveals energy eigenvalue E_ℓ of $\hat{H}_{\text{spin}}$,
- prepares system in energy eigenstate $|\ell\rangle$
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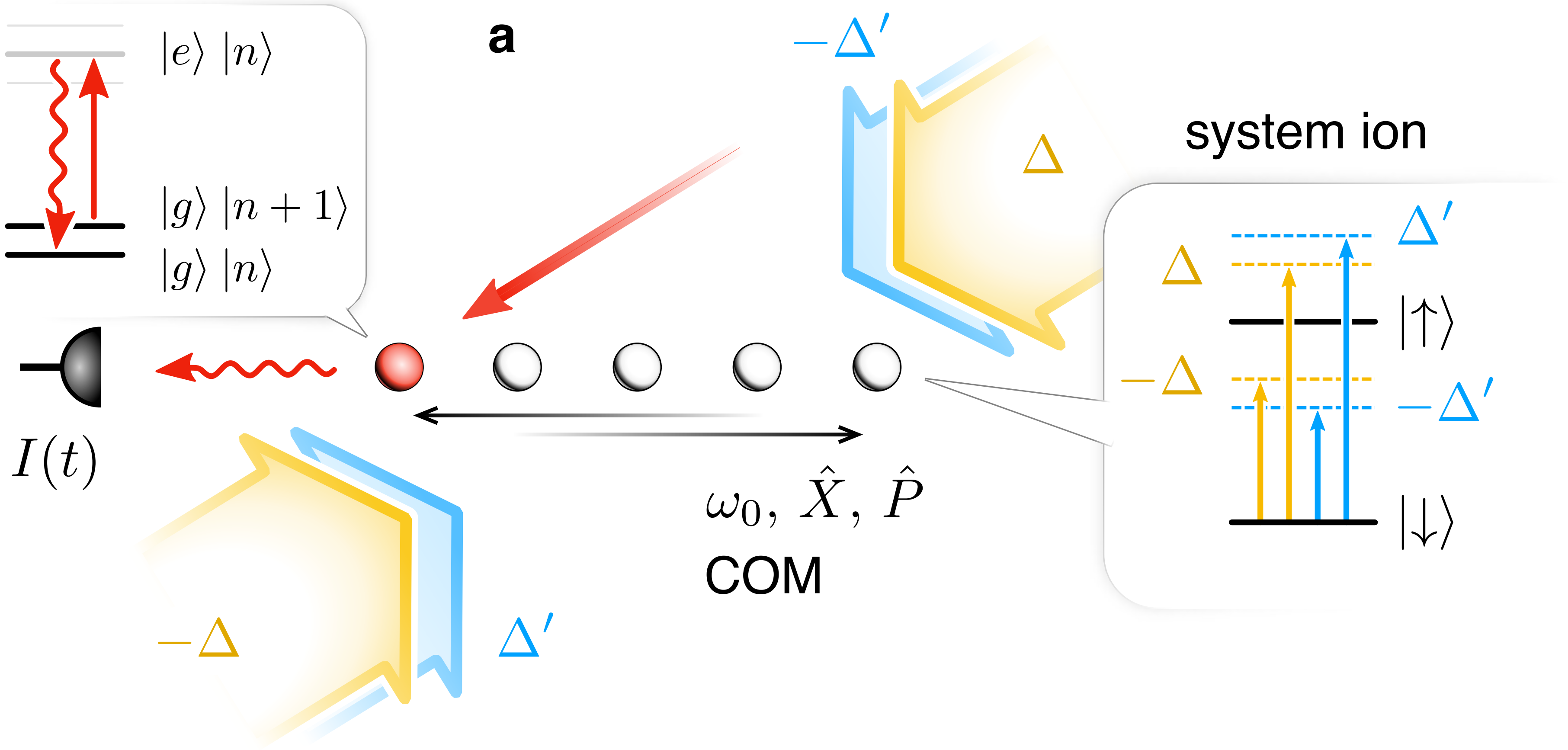
Trapped-Ion Implementation of the QND gate

Trapped-Ion Implementation of QND measurement

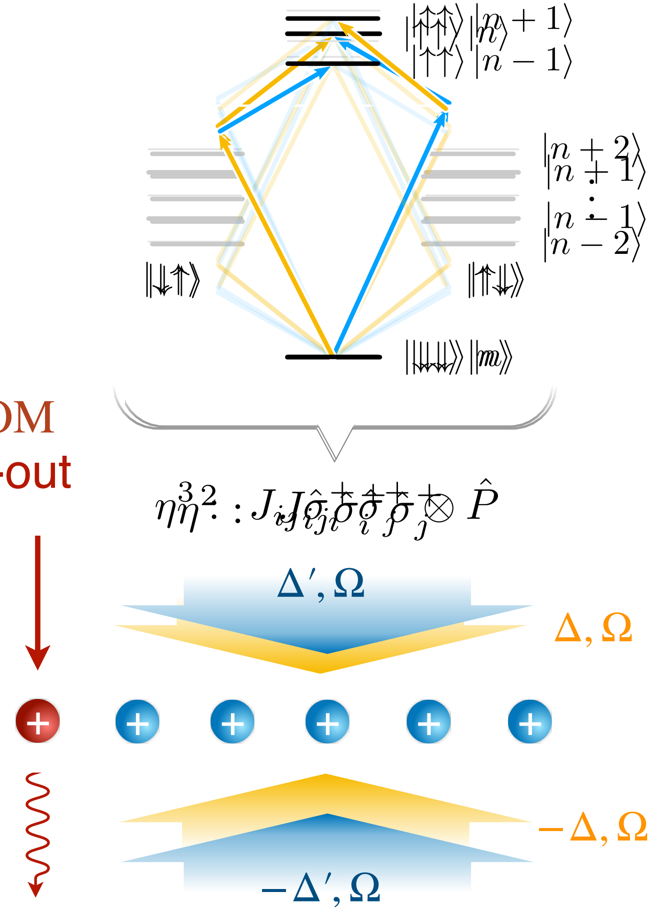


Full system Hamiltonian of Double Mølmer-Sørensen configuration:

$$\hat{H}_{\text{int}} = \frac{\Omega}{2}\sum_n \hat{\sigma}_n^+\left(\underline{e^{-i\Delta t+ik\hat{Z}_n}} + [1+\frac{\delta\Omega}{\Omega}]\underline{e^{i\Delta t-ik\hat{Z}_n}} + \underline{e^{-i\Delta' t-ik\hat{Z}_n}} + [1+\frac{\delta\Omega}{\Omega}]\underline{e^{i\Delta' t+ik\hat{Z}_n}}\right) + h.c.$$

Trapped-Ion Implementation of QND measurement

$\hat{P}_{\mathrm{COM}}$ read-out



homodyne

\`double'
Molmer-Sørensen interaction

D. Porras and J. I. Cirac
Phys. Rev. Lett. **92**, 207901 (2004)

Hamiltonian for system + meter

$$\boxed{H_{\mathcal{S}\mathcal{M}} = \cancel{\hat{H}'_{\mathrm{spin}}} + \vartheta(t)\, \hat{H}_{\mathrm{spin}} \otimes \hat{P}_{\mathrm{COM}}}$$

$\hat{H}' = \hat{H}$ QND !

fine tuning

here: weak coupling

transverse Ising model

$$\hat{H}'_{\mathrm{spin}} = -\sum_{i<j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + h' \sum_{j=1}^{N} \hat{\sigma}_j^z$$

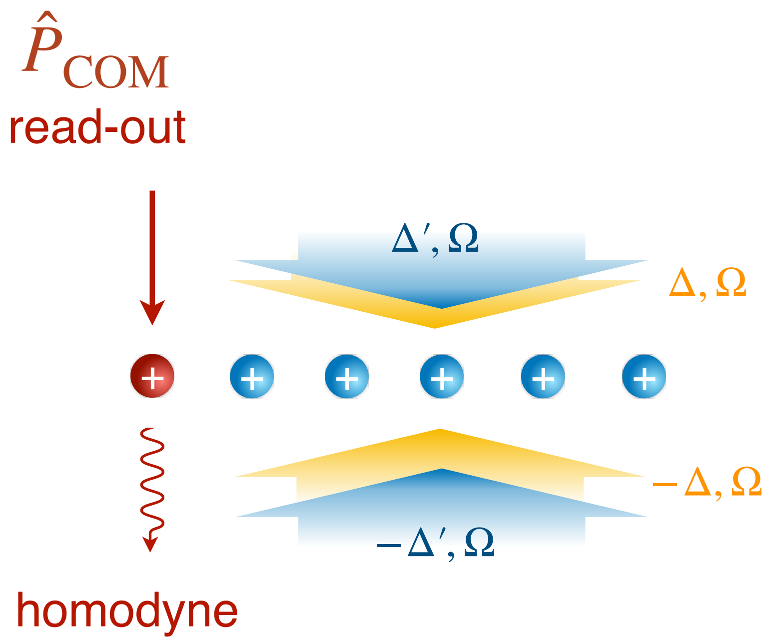
$$\hat{H}_{\mathrm{spin}} = -\sum_{i<j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + h \sum_{j=1}^{N} \hat{\sigma}_j^z$$

we tune: $\Delta' - \Delta = \nu_{\mathrm{COM}} \longrightarrow \hat{P}_{\mathrm{COM}}$ meter

trap frequency

Stochastic Density Matrix Equation

Simulating single run of homodyne measurement



$$d\hat{\rho}_c(t) = -i[\hat{H}', \hat{\rho}_c(t)]dt + \gamma\mathcal{D}[\hat{H}/J]\hat{\rho}_c(t)\,dt + \sqrt{\gamma\epsilon}\,\mathcal{H}[\hat{H}/J]\hat{\rho}_c(t)\,dW(t)$$

$$dX(t) \equiv \boxed{I(t)}dt = 2\sqrt{\gamma\epsilon}\boxed{\langle\hat{H}/J\rangle_c}dt + dW(t)$$

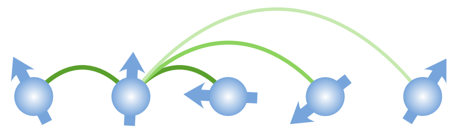
The homodyne current provides a continuous readout of the energy of the quantum many-body system

---

$$\mathcal{D}[\hat{s}]\hat{\rho}_c \equiv \hat{s}\hat{\rho}_c\hat{s}^\dagger - (\hat{s}^\dagger\hat{s}\hat{\rho}_c + \mathrm{H.c.})/2$$

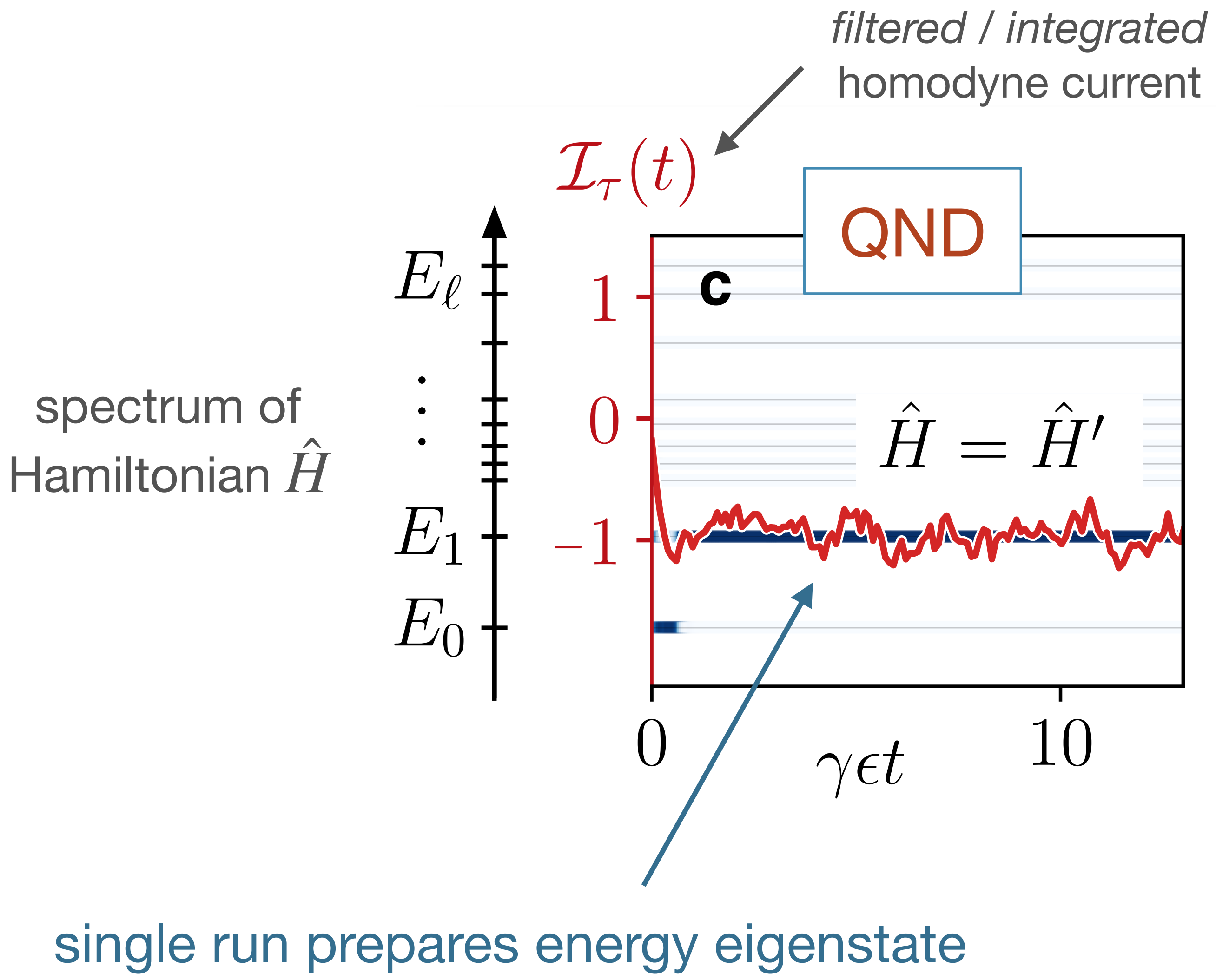
$$\mathcal{H}[\hat{s}]\hat{\rho}_c \equiv (\hat{s} - \langle\hat{s}\rangle_c)\hat{\rho}_c + \mathrm{H.c.}$$

QND Measurement of $\hat{H}_{\text{spin}}$- *a Single Run*



$N = 5$ spins, $\alpha = 1.5$, $h/J = 1.5$.

Homodyne current from scattered light reveals `energy'



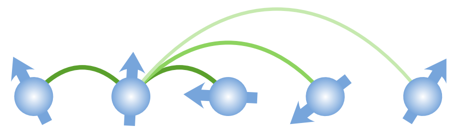
- homodyne current reads energy

$$\mathcal{I}_\tau(t) \sim \langle \hat{H}_{\text{spin}} \rangle_c + \text{noise}$$

- we illustrate single runs of an experiment by simulating a stochastic density matrix eq. for homodyne measurement

QND Measurement of $\hat{H}_{\text{spin}}$ - *a Single Run*

Homodyne current from scattered light reveals `energy'



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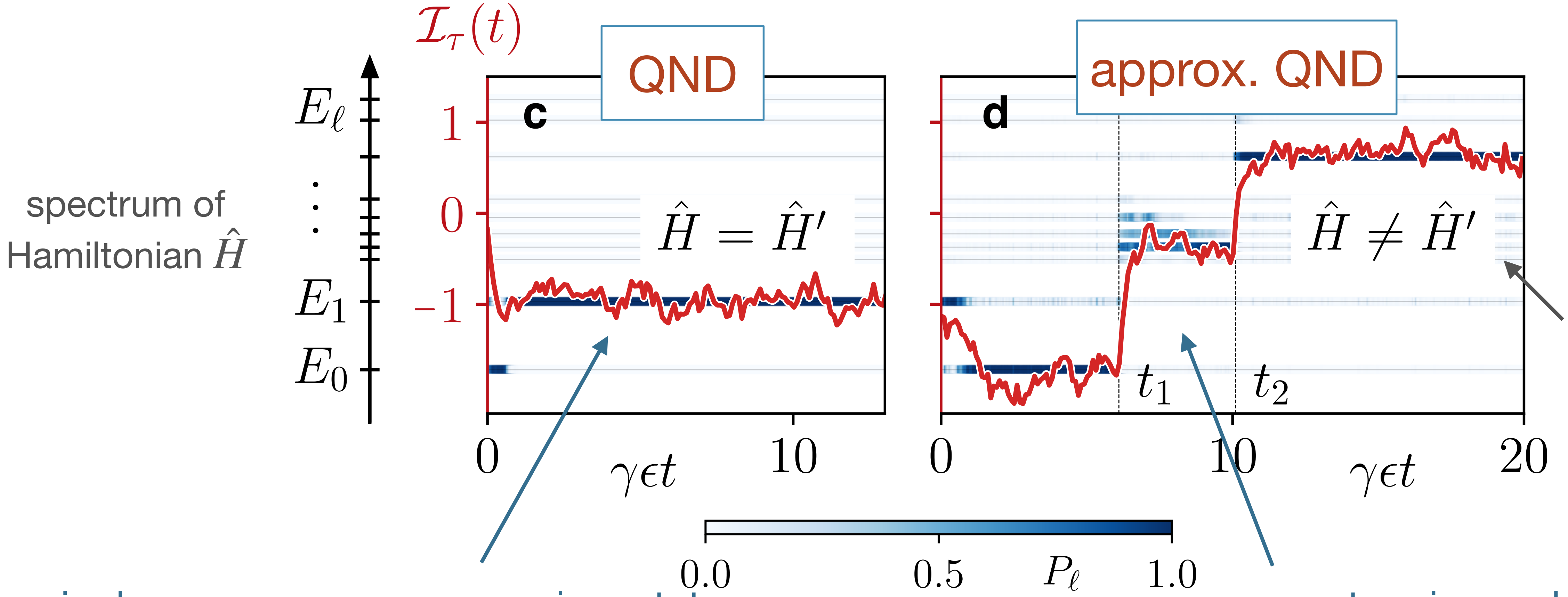


$$\hat{H}' = \hat{H} + \delta\tilde{h}\sum_j \hat{\sigma}_j^z$$

$$\delta\tilde{h}/J = -0.2$$

single run prepares energy eigenstate

quantum jumps between eigenstates

QND Measurement of Many-Body Hamiltonian - *Single Run*

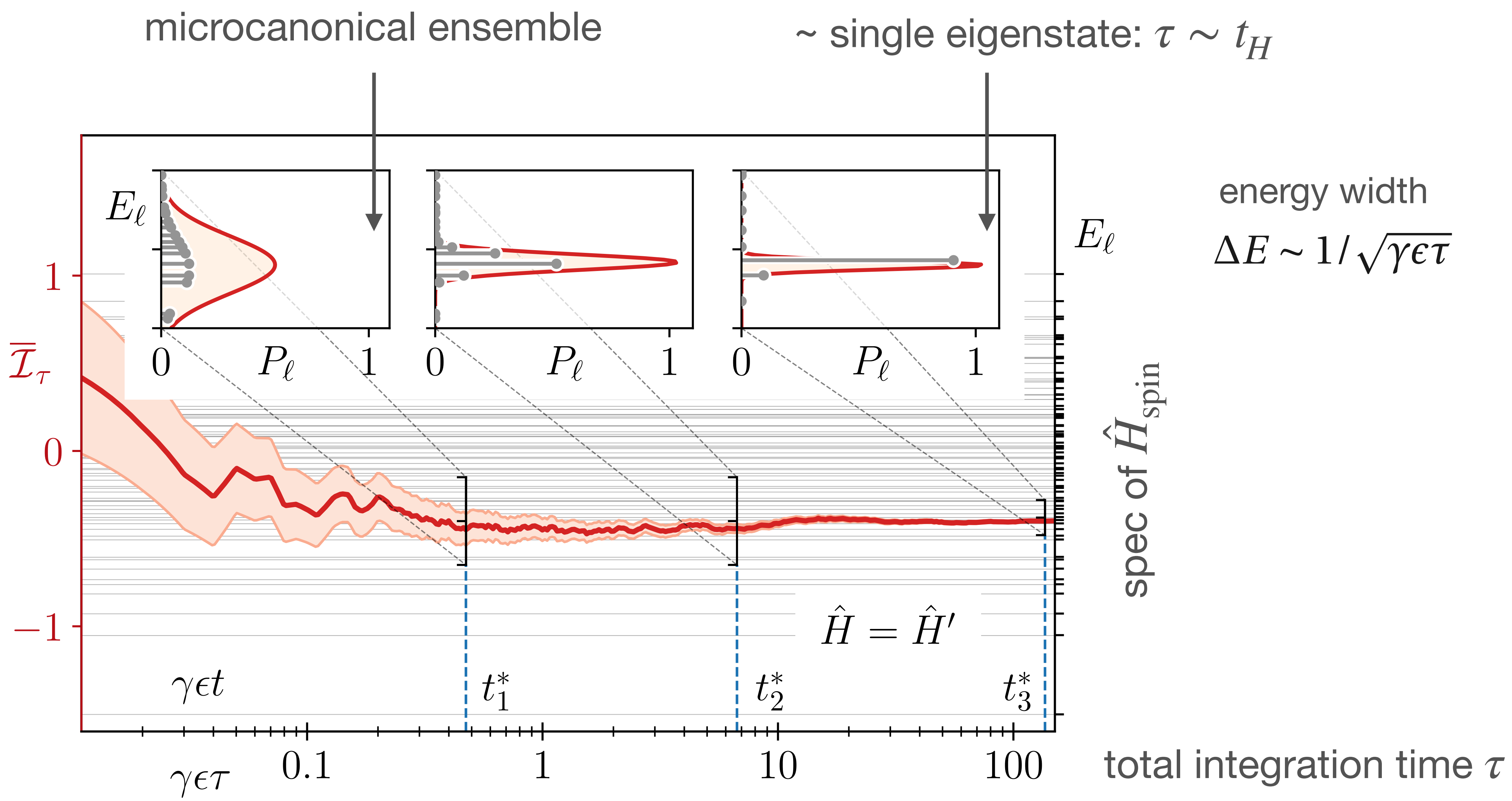
N=8 spins

time integrated homodyne current

$\overline{\mathcal{I}}_\tau = (2N\sqrt{\gamma\epsilon\tau})^{-1}\int_0^\tau I(t)dt$

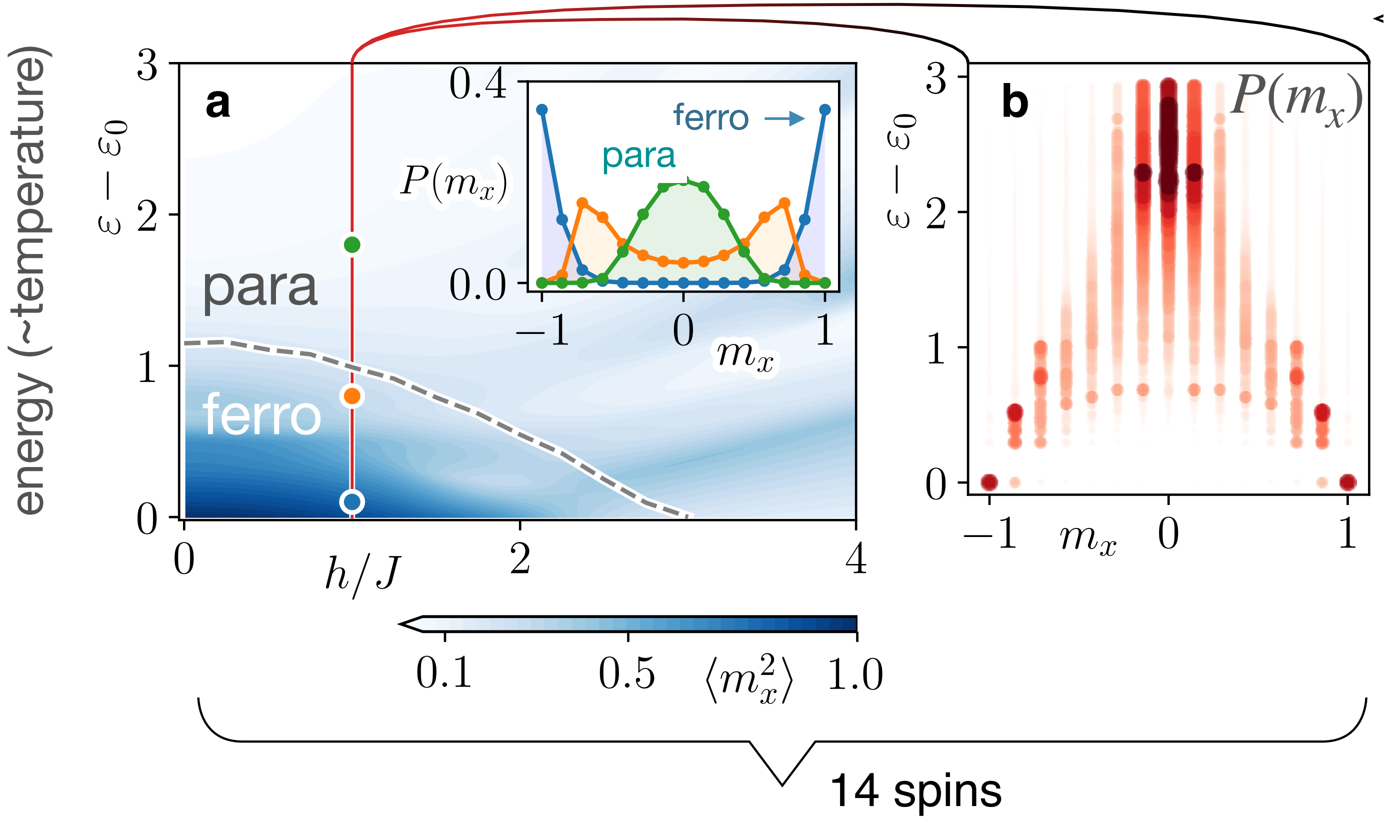
energy width

$\Delta E \sim 1/\sqrt{\gamma\epsilon\tau}$



Eigenstate Thermalization Hypothesis (for mesoscopic systems)

Excited-state phase transition in the Ising model with $J_{ij} \sim J/|i-j|^{\alpha=1.5}$



microcanonical ensemble $\Delta E/(JN) = 0.1$

- ETH Keith R. Fratus, Mark Srednicki, arXiv:1611.03992

$$\langle \ell | \hat{O} | \ell \rangle = O(E_\ell) = \mathrm{tr}(\hat{O}\hat{\rho}^{\mathrm{mc}}_{E_\ell})$$

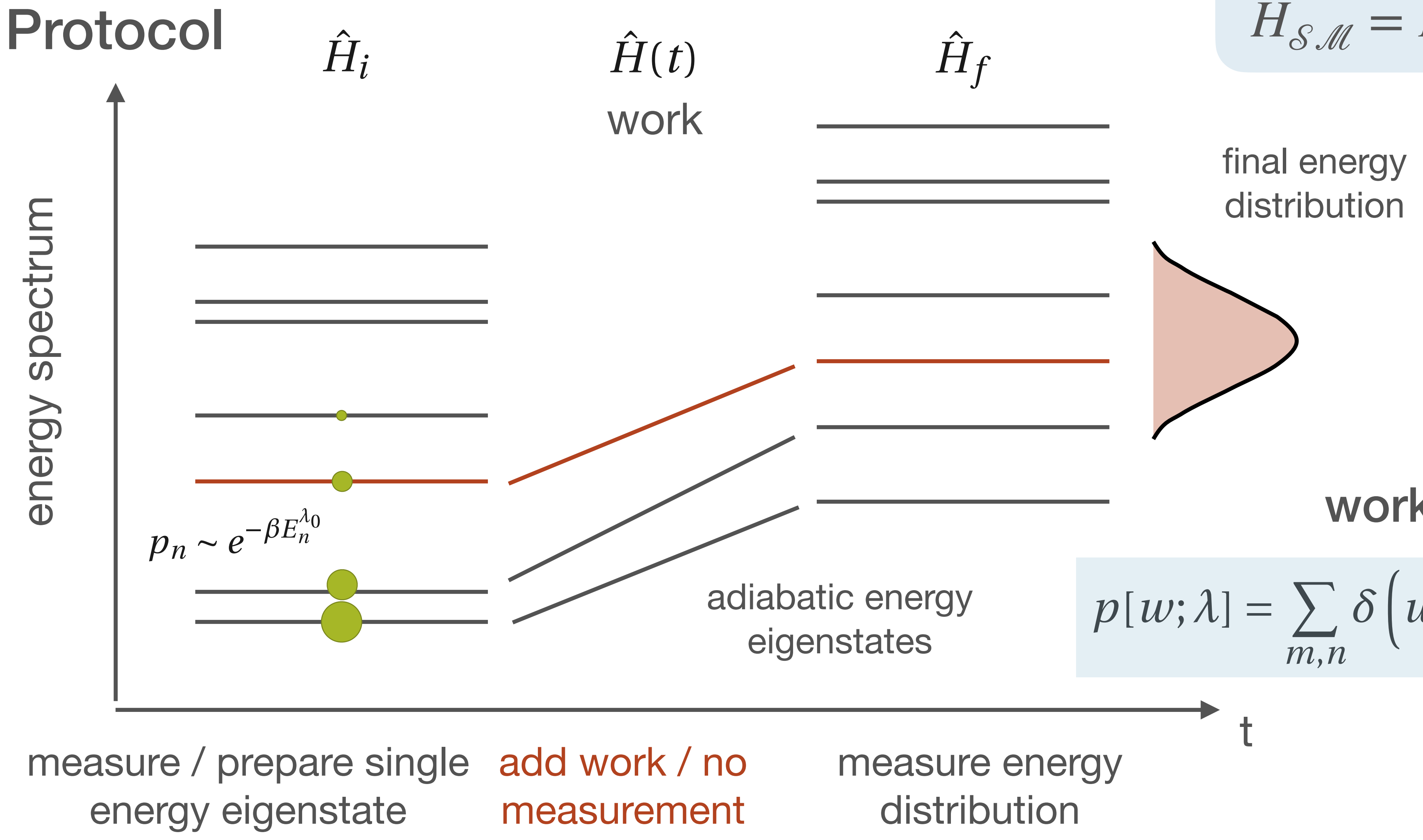
$$\langle \ell' \mid \hat{O} \mid \ell \rangle = O(\bar{E})\delta_{\ell'\ell} + e^{-S(\bar{E})/2} f_{\hat{O}}(\bar{E}, \omega) R_{\ell'\ell}$$

- ferro-paramagnetic transition for for $1 < \alpha \leq 2$ manifest in $P(m_x)$ or $\langle \hat{m}_x^2 \rangle$ with

$$\hat{m}_x = N^{-1} \sum_j \hat{\sigma}_j^x$$ magnetization

Measuring Work Distribution in Quantum Thermodynamics

$$H_{\mathcal{S}\mathcal{M}} = \hat{H}'_{\text{spin}} + \vartheta(t)\,\hat{H}_{\text{spin}} \otimes \hat{P}_{\text{COM}}$$



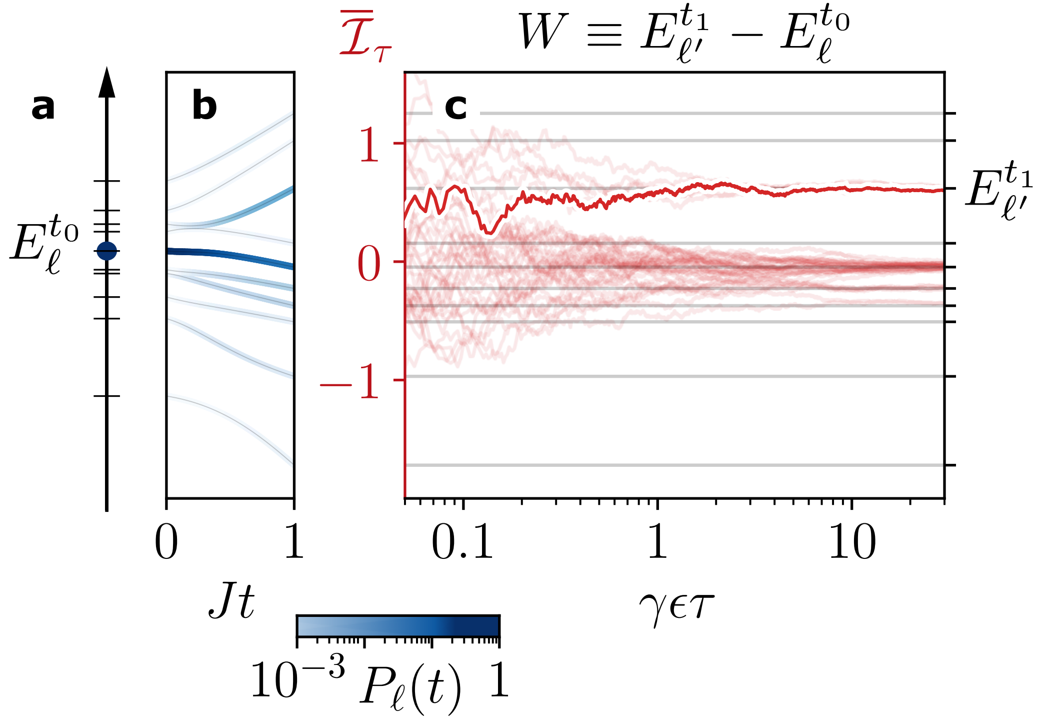
work distribution

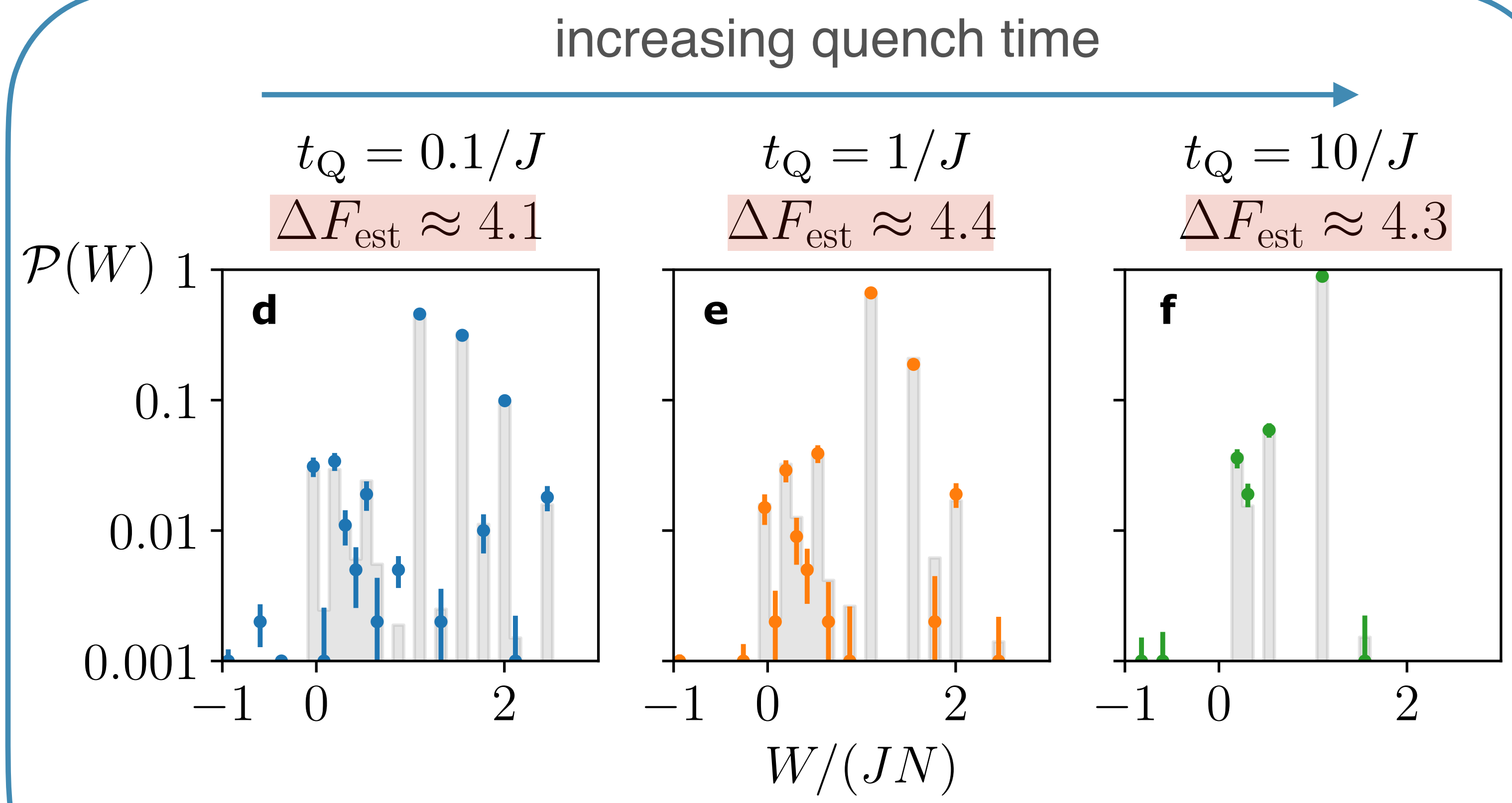
$$p[w;\lambda] = \sum_{m,n} \delta\left(w - \left[E_m^{\lambda_\tau} - E_n^{\lambda_0}\right]\right) p_{m|n}[\lambda]\, p_n^0$$

Work distribution function and quantum fluctuation relations (QFRs)

Verification of Jarzynski equality $\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F}$ with 5 spins

Protocol





1000 simulated experimental runs

first potential test of QFR in quantum many-body

Quantum fluctuation relations: Foundations and applications, M Campisi, P Hänggi, and P Talkner, RMP 2011

Conclusions

QND gate — Applications

QND gate $\mathscr{U}_{\mathrm{QND}}$ is a building block of quantum algorithms like quantum phase estimation (QPE) which enables

$$\mathscr{U}_{\mathrm{QND}} = e^{-i\hat{H}_{\mathrm{QND}}t}$$

- Measurement of many-body Hamiltonian $\hat{H}_{\mathrm{spin}}$:

This talk

  - Test of Eigenstate Thermalisation Hypothesis
  - Work distribution function and Quantum Fluctuation Relations

$$\hat{H}_{\mathrm{QND}} = \hat{H}_{\mathrm{spin}} \otimes \hat{P}_z$$

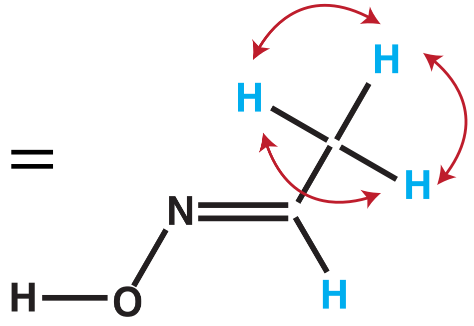
D. Yang, et. al.
Nat. Commun. **11**, 775 (2020)

- Study of energy level statistics of quantum many-body systems via Spectral Form Factor (SFF)

$$\hat{H}_{\mathrm{QND}} = \hat{H}_{\mathrm{spin}} \otimes \hat{\sigma}_z$$

DV et.al.
PRX Quantum 1, 020302 (2020)

- Sampling many-body spectral functions $\hat{H}_{\mathrm{spin}} =$



D. Sels, E. Demler
arXiv:1910.14213 [quant-ph]

- …

Thank you